Indian J. Phys. **77B** (4), 469-471 (2003)

Note

AN INTERNATIONAL JOURNAL IJP B

On oscillatory Bénard-Marangoni convection in a fluid layer with strong surface deformation

Ishak Hashim

School of Mathematical Sciences, Faculty of Science & Technology, National University of Malaysia, 43600 UKM Bangi, Selangor, Malaysia

E-mail : ishak_h@pkrisc.cc ukm my

Received 7 March 2003, accepted 24 March 2003

Abstract : Linear stability theory is applied to the problem of the onset of oscillatory Bénard-Marangoni convection in a horizontal layer of fluid bounded from below by a rigid boundary and above by a strongly deformable free surface and heated from below. We obtain critical values of the Prandtl number below which convection sets in as oscillatory motions and give an example of a situation in which two different modes of instabilities coexist.

Keywords : Bénard-Marangoni convection, linear stability theory

PACS Nos. : 47.20.Bp, 47.20.Dr, 47.27.Te

The onset of convection driven by both buoyancy (Bénard) and surface-tension-driven (Marangoni) effects in initially quiescent, unbounded, horizontal fluid layer was first analysed theoretically by Nield [1]. Nield's linear stability analysis concerned only the case of a non-deformable free surface and this restriction was subsequently relaxed by Davis and Homsy [2], who found that surface deformation stabilises Bénard convection but destabilises Marangoni convection. Both these contributions concerned the onset of steady convection. The possibility of oscillatory convection was first addressed by Gouesbet *et al* [3] who concluded on the basis of an extensive numerical search that oscillatory convection is impossible when the layer is heated from below. All these authors assumed that the two driving mechanisms (represented by the non-dimensional Rayleigh and Marangoni numbers respectively) were independent. However, this may not be the case in practice and Benguria and Depassier [4] and Hashim and Wilson [5] addressed the problem in the more physically-relevant case in which the Rayleigh and Marangoni numbers are linearly dependent and found that oscillatory convection is now possible when the layer is heated from below if the free surface is deformable.

The linearised equations and boundary conditions governing the onset of Bénard-Marangoni in an initially quiescent horizontal fluid layer bounded above by a deformable free surface and bounded below by a thermally conducting planar boundary subject to a uniform temperature gradient have been obtained by several authors (see, for example, Benguria and Depassier [4] and Hashim and Wilson [5]) and are given by

$$(D^2-a^2)\left(D^2-a^2-\frac{s}{P_r}\right)w-a^2RT=0, \tag{1}$$

$$\left(D^2-a^2-s\right)T+w=0, \tag{2}$$

subject to

$$sf-w=0, \tag{3}$$

$$\left(D^2-3a^2-\frac{s}{P_r}\right)Dw-a^2\left(P_rG+\frac{a}{C_r}\right)f=0. \tag{4}$$

$$(D^2+a^2)w+a^2M(T-f)=0, \tag{5}$$

$$DT+B_i(T-f)=0, \tag{6}$$

evaluated on the undisturbed position of the upper free surface $z=1$, and

$$w=0, \tag{7}$$

$$Dw = 0, \tag{8}$$

$$T = 0, \tag{9}$$

evaluated on the lower rigid boundary $z = 0$, where the operator $D = d/dz$ denotes differentiation with respect to the vertical coordinate z. The variables $w = w(z)$, $T = T(z)$ and f denote the vertical variation of the z-velocity and temperature and the magnitude of the free surface deflection of the linear perturbation to the basic state with total wave number a in the horizontal x–y plane and complex growth rate s with $\mathrm{Im}(s) = w$. The non-dimensional groups appearing in the problem are the Rayleigh number R, the Marangoni number M, the Prandtl number P_r, the capillary number C_r, the Galileo number G and the Biot number B_i as defined in Hashim and Wilson [5]. The Rayleigh number R and the Marangoni number M are related by $M = \Gamma R$ where Γ is constant and so R and M are linearly dependent. The eigenvalues ω and R depend on a, P_r, Γ, B_i and the group $(P_rG + a^2/C_r)$. In the limit $G \to \infty$ and/or $C_r \to 0$, the free surface is nondeformable. Following Benguria and Depassier [4], we set $1/C_r = 0$, *i.e.* the effect of surface deformation is determined only by G. Benguria and Depassier [4] first identified numerically the oscillatory marginal curves in this case, although in all the examples they described that the onset of convection was steady. In this note, we present an example of a situation in which oscillatory convection is preferred to steady convection and obtain critical values of the Prandtl number below which this situation occurs.

In Figure 1 we plot marginal stability curves for a range of values of P_r and G satisfying $P_rG = 250$.

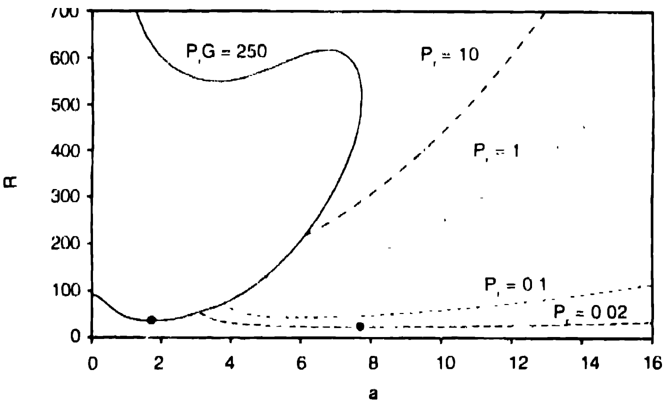

Figure 1. Numerically-calculated marginal stability curves (steady - solid line, oscillatory - dashed lines) in the case $\Gamma = 1.8$ and $B_i = 0$ for a range of values of P_r and G satisfying $P_rG = 250$. The global minima are marked with dots (•)

The inside and outside of the steady marginal curve are the regions of instability and stability with respect to steady convection respectively, while the regions above and below the oscillatory marginal curve are the regions of instability and stability with respect to oscillatory convection respectively. In the case shown in Figure 1, the oscillatory marginal curve, first identified by Benguria and Depassier [4] leaves the steady one at a non-zero value of a and extends to $a \to \infty$, while the steady one only extends to a finite value of a. Figure 1 also shows that the onset of convection can be either steady or oscillatory, and in the case shown the minima of the steady and oscillatory marginal curves are equal when $P_r = P_{rc} = 0.0653$, corresponding to competition between two different modes at the onset of convection. Steady convection is preferred for $P_r > P_{rc}$ and oscillatory convection for $P_r < P_{rc}$. Unfortunately, Benguria and Depassier [4] failed to find a case in which oscillatory convection is preferred. However, very recently Hashim [6] has shown analytically in the limit of very short waves, $a \to \infty$, that the effect of decreasing the Prandtl number is to decrease the oscillatory marginal curves, indicating the possibility of oscillatory motions as the primary instability. In Table 1 we present the numerically-calculated values of P_{rc} (critical values below which convection sets in as oscillatory motions) for a range of values of P_rG and Γ in the case $B_i = 0$.

Table 1. Numerically-calculated critical values P_{rc} below which oscillatory motion is the preferred mode of instability for a range of values of (a) P_rG with fixed $\Gamma = 1.8$ and (b) Γ with fixed $P_rG = 250$ both in the case $B_i = 0$

(a)

P_rG	P_{rc}	a_c	R_c	ω_c
50	0.1435	3.532	17.521	2.833
150	0.1174	4.993	31.797	5.295
250	0.0653	6.331	35.796	6.283
1000	0.0077	15.295	40.265	7.560

(b)

Γ	P_{rc}	a_c	R_c	ω_c
0.5	0.0734	6.037	131.698	6.476
1	0.0675	6.247	64.816	6.335
1.8	0.0653	6.331	35.796	6.283
5	0.0638	6.394	12.830	6.244

In this case, the effect of increasing P_rG is to decrease the critical value P_{rc} and for a fixed P_rG, the effect of increasing Γ is also to decrease P_{rc}. We have shown in this note that there are many situations in which oscillatory convection is the primary instability and competition between modes exists in the problem considered by Benguria and Depassier [4].

Acknowledgment

The author acknowledges valuable assistance from Prof. Dr S. K. Wilson of Mathematics Department, University of Strathclyde, UK. The financial support from Universiti Kebangsaan Malaysia is gratefully acknowledged.

References

[1] D A Nield *J Fluid Mech* **19** 341 (1964)

[2] S H Davis and G M Homsy *J. Fluid Mech.* **98** 527 (1980)

[3] G Gouesbet, J Maquet, C Rozé and R Darrigo *Phys Fluids* **A 2** 903 (1990)

[4] R D Benguria and M C Depassier *Phys Fluids* **A 1** 1123 (1989)

[5] I Hashim and S K Wilson *Int J Engg Sci* **37** 643 (1999)

[6] I Hashim *Indian J Phys* **76B** 605 (2002)